An Examination of Grade Three Mathematics Textbooks for Problem Solving Strategies

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Abstract

Three grade three mathematics textbooks were selected arbitrarily (every other) from a total of six currently used in the schools of Ontario. These textbooks were examined through content analysis in order to determine the extent (i.e., the frequency of occurrence) to which problem solving strategies appear in the problems and exercises of grade three mathematics textbooks, and how well they carry through the Ministry's educational goals set out in The Formative Years.

Based on Polya's heuristic model, a checklist was developed by the researcher. The checklist had two main categories, textbook problems and process problems and a finer classification according to the difficulty level of a textbook problem; also six commonly used problem solving strategies for the analysis of a process problem. Topics to be analyzed were selected from the subject guideline The Formative Years, and the same topics were selected from each textbook. Frequencies of analyzed problems and exercises were compiled and tabulated textbook by textbook and topic by topic. In making comparisons, simple frequency count and percentage were used in the absence of any known criteria available for judging high

or low frequency. Each textbook was coded by three coders trained to use the checklist.

The results of analysis showed that while there were large numbers of exercises in each textbook, not very many were framed as problems according to Polya's model and that process problems form a small fraction of the number of analyzed problems and exercises. There was no pattern observed as to the systematic placement of problems in the textbooks.

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CHAPTER ONE

INTRODUCTION TO THE STUDY

Developing skill in problem solving has long been recognized as one of the important goals in elementary school mathematics. The place of problem solving in mathematics is emphasized in The Formative Years, the curriculum document put out by the Ministry of Education, and intended to be the guide to curriculum in the elementary schools of Ontario. It asks that schools provide programs for the child to develop competence to draw conclusions from evidence obtained through experimentation or logical reasoning and apply mathematics to the solution of everyday practical problems[1] and to apply arithmetic to everyday problems and recognize through such activities the need for further skills.[2] The Ministry states that the objective of teaching problem solving is to acquire the basic skills fundamental to a continuing education, specifically the ability to apply rational or intuitive processes to the identification, consideration and solution of problems. Each individual should develop skills of inquiry, analysis, synthesis, and evaluation. Children who acquire such reasoning skills will be able to continue learning throughout their lives.[3] The Ministry also recognizes that mathematics does not belong just in the mathematics classroom but:

> the concepts and skills of mathematics, combined with its vocabulary, help the child to discriminate, classify, and think logically. They are developed and reinforced by observation, imitation, discussion, inquiry, investigation, experimentation, practice, and application in day-to-day activities, both in school and out.[4]

The recommendation from the National Council of Teachers of Mathematics (NCTM) that problem solving be the focus of school mathematics in the 1980's is the first of eight recommendations expressed in An Agenda For Action: Recommendations for School Mathematics of the 1980's.

Problem solving is underscored as the heart of school mathematics, a skill to be nurtured and encouraged. This position statement of the National Council of Supervisors of Mathematics (NCSM, 1977) emphasizes the development of problem solving abilities for students of all achievement levels.

In Ontario this stated emphasis on problem solving is reflected in the curriculum guidelines developed by the local Boards of Education. The Peel Board of Education recognizes problem solving as:

> far more than a mathematics skill. It is a life skill, that should be taught across the curriculum. Students should recognize a problem situation

> any time it happens, at school (any subject) or at home.[5]

The Hamilton Board of Education emphasizes the importance of problem solving by stating:

> One area of great importance in the curriculum and throughout the whole of mathematics instruction is that of problem solving, and that priority must be given to this aspect of mathematics.[6]

The inclusion of problem solving strategies in the curriculum helps a child meet real-life experiences later on. This is one of the objectives of teaching problem solving put forth by the Toronto Board of Education.

According to Peterborough County Board of Education, learning a repertoire of problem solving strategies and techniques:

> changes a large variety of problems from the insoluble (or soluble with difficulty) to the relatively routine category. This seems to be a desirable outcome of problem solving instruction, particularly if the problems which can be solved using this repertoire are similar to those which will be frequently encountered outside the school.[7]

In order to face the ever increasing multitude of technological, ecological and sociological problems facing mankind, our schools must nurture and stimulate the students to think independently and creatively. The mathematics class affords us the opportunity to foster and encourage this development. This is the position and approach of the Halton Board of Education regarding problem solving.

Moving on from this position, it becomes obvious that elementary school is responsible for teaching basic mathematical principles reflected in the degree to which students' problem solving skills are developed.

In order to meet these objectives, most elementary schools make extensive use of mathematics textbooks. The text, therefore, should show cognizance of the mathematics objectives stated by the many Boards of Education if such objectives are to be achieved through in-school introduction. Most textbooks start formal development of problem solving skills at the grade three level; hence the reason for choosing this grade for the study.

Need for the Study

Education decision makers should want to know what congruence there is between problem solving strategies, employed in the problems and exercises of the mathematics textbooks they use, and the emphasis given to the educational objectives of problem solving in their directives. Among these decision makers are the textbook writers; curriculum writing teams; Ministry of Education personnel who put the curriculum together; Board personnel who decide which books and what parts of the curriculum should be emphasized; practitioners such as the teachers who are trying to put into practice the curriculum guidelines in order to fulfil the objectives for the development of problem solving skills. This research should provide some relevant information for these users identified.

Statement of the Problem

Grade three mathematics textbooks will be examined through content analysis in order to determine the extent to which problem solving strategies appear and how well they carry through the Ministry's educational goals of problem solving as set out in The Formative Years.

Purpose of the Study

The major goals of this study are to identify problem solving strategies contained in the exercises and problems of grade three mathematics textbooks that are selected from "Circular 14 Textbooks 1984", using heuristics or strategies derived from Polya's model; to determine the extent (i.e., the frequency of occurrence) to which these identified problem solving strategies are represented in the exercises and problems of grade three mathematics textbooks; and examine and compare, textbook by textbook and topic by topic the increasing or decreasing frequencies and proportions of the exercises and problems contained in each of the identified problem solving strategies in grade three mathematics textbooks.

Limitations of the Study

1. This study demonstrates the use of a particular model for describing and analyzing the problem solving strategies employed in grade three mathematics textbooks. There are other problem solving models that could be applied for the same purpose at any grade level; so the use of one model and one grade delimits this research for those who may approach the same study with a different technique and in different grades or across grade levels.
2. This research is limited to the analysis of grade three mathematics textbooks. No ancilliary material was analysed.
3. Only fifty percent of the sample textbooks, listed in "Circular 14" was analysed.

Operational Definitions

Mathematical Problem:

A mathematical problem is a mathematical situation which poses a question in light of some data. The individual attempting to answer the question does not possess an immediate solution. Hence the act of solving a mathematical problem requires prior knowledge of mathematical concepts, and a repertoire of problem solving strategies.

Exercise:

Any mathematical sentence without words. For example:

(i) Write the missing numbers:

2, 4, 6, ___, ___, ___.

10, 18, ___, 38, ___, ___.

20, 30, 40, ___, ___, ___, ___.

(ii) 408 $\begin{matrix} + \\ - \\ \times \\ \div \end{matrix}$ 34 =

Problem:

Any mathematical sentence that uses words in addition to symbols or numerals. For example:

A sporting goods store has 150 footbals worth $2.00 each and 75 softballs worth $2.50 each. What is the total value of the footballs and softballs on hand?

Problem Solving Strategies:

The strategy is the plan or method used for solving a mathematical problem. The problem might be solved by using more than one method. The strategies used in this study derive from George Polya's four-step model.

<u>Circular 14 Textbooks</u>:

This is an annual publication that provides a list of texts approved by the Ministry of Education, under the Educational Act, for use in Ontario schools.

The Formative Years:

The Formative Years is a curriculum document published under the authority of the Minister of Education. It sets out the goals for the Primary and Junior Divisions of the public and separate schools of Ontario and states the expectations of the Ministry of Education with regard to the programs developed at the local level to meet these goals.

Summary

Chapter One presents an introduction to the study undertaken with support for the development of problem solving skills as presented in curriculum documents of the Ministry and local Boards of Education. Need for the research giving its limitations and operational definitions are also stated.

Footnotes - Chapter One

1. Ministry of Education, Ontario. The Formative Years (PIJI), 1975, p. 11.

2. Ibid., p. 6.

3. Ministry of Education, Ontario. Education in the Primary and Junior Divisions, 1975, p. 6.

4. Ibid., p. 62.

5. Peel Board of Education. Problem Solving in Level Four Mathematics Core Review, p. 2.

6. Hamilton Board of Education. Mathematics: Elementary Primary Division, p. 83.

7. Peterborough County Board of Education. Computational Problem Solving: Junior Division, p. 3.

CHAPTER TWO

PROBLEM SOLVING STRATEGIES

Historical Perspectives on Problem Solving Strategies

Problem solving has been a subject of research by educators, educational psychologists, mathematicians, and philosophers since the early 1900's.

It has been studied in one form or another and the proponents of various theories have attempted to explain problem solving phenomena in terms of their own theoretical constructs. Despite the theoretical diversity there exists a consensus that a"problem" refers to a situation in which an individual is called upon to perform a task not previously encountered and for which externally provided instructions do not specify completely the mode of solution (Resnick, 1976).

Duncker: Karl Duncker (1945), the spokesman for the Gestalt psychology, defines "problem" as a situation that arises when there is a goal, but the method for reaching the goal is unknown. He suggests that solution to any problem is attained by restructuring the problem. There are solution phases and each successive phase refines the previous one until some concrete evidence possessing the characteristics of a solution is reached. There is a shifting back and forth among the phases until a "productive reformulation" of the original problem is attained.

Duncker presents a "family tree" for the radiation problem in which each solution phase is more specific than the one above it, but more general than the one below. Each solution phase corresponds to a ground of conflict present in the situation. So analysis of the situation is, therefore, primarily analysis of the goal. Each goal has sub-goals which are continually being refined, and during the process differences are being reduced. This, in its turn, depends on the individual's capacity of recall, memory search, and suitable environmental conditions.

Wertheimer: Max Wertheimer (1959), like Duncker, views problem solution as a result of re-orientation and reorganization of the problem situation. The emphasis in the Gestalt analysis is on the "insightful" nature of the process or more penetrating perspective of the situation. These changes of the situation as a whole imply changes in the structural meaning of part items, changes in their place, role and function that often lead to important consequences.

Duncker is more explicit than Wertheimer in suggesting ways in which analyses of the demands of a problem can lead to a solution. However, Gestalt views presented here do not give any explicit or specific enough strategy for problem solving that could be applied in the school curriculum.

Dewey: John Dewey (1938) evolved a theoretical model of "scientific inquiry" that was applicable in the public school environment. According to his theory, every individual is believed to be a dynamic problem solving organism that can be taught to organize his problem solving skills into flexible habits of thought and behaviour.

Problem solving according to Dewey is "the directed and controlled transformation of an indeterminate situation into a determinately unified one."[1] The pattern of inquiry has a common structure that denotes a difference in the ways people carry out their inquiries and the ways in which they "ought" to carry on their activities. This pattern when applied to an indeterminate situation where the elements do not hang together leads by a step-by-step activity to a determinate situation which is controlled and finished.

The steps according to Dewey's theory of scientific inquiry are as follows:

1. Problem is Felt: When there is a problematic situation, there is a confusion that leads to an emotional reaction. This interaction with the situation becomes inquiry when facts of the case and their potentialities are observed.

2. Problem Is Located and Defined: A problem well put is half answered. The conditions that cause the problem are re-inspected and stated in some degree on the basis of observed facts. The difficulty is getting located and defined. It is becoming a true problem, something intellectual. The conditions that constitute the trouble and cause the blockage of action are noted more definitely. This is the most difficult step in problem solving.
3. Possible Solutions Are Suggested: The degree to which a problem is defined suggests ideas for the kind of solution(s) that is(are) needed. The observed facts of the case or the data set the problem before us, and insight into the problem corrects, modifies and expands the suggestion that originally occurred. In this fashion, the suggestion becomes a definite supposition or stated more technically, a hypothesis. The intellectual element consists in what we do with the hypothesis and how we use it.
4. Reasoning: Reasoning has the same effect upon a suggested solution that more intimate and extensive observation has upon the original trouble. A thorough review of the suggestion, which seemed plausible at first sight, is done, tracing its full consequences, until an idea is reached which can investigate and direct the inquiry. This leads to the

acceptance or rejection of the hypothesis. Sometimes modifications are made so that the hypothesis suits and organize the facts of the case. Reasoning helps extend knowledge, while at the same time it depends upon the store of knowledge, prior experience, and special education of the individual who is carrying on the inquiry.

5. <u>Evaluation</u>: In the course of this process described so far, facts of the case are selected and arranged in definite ways until they produce a definite result when all relevant data have been gathered. The question is how the data is to be interpreted, estimated, appraised, and placed.

   This issue is defined by:

   (a) the determination of the data that are important in the given case, and

   (b) the elaboration of the conceptions and meanings suggested by the crude data.

The sequence of the five steps presented above is not fixed. They are indispensable traits in the work of inquiry, but do not follow each other in a set order. In practice, two of them may telescope; any two or more of them may be passed over hurriedly. The burden of reaching a conclusion may fall on a single step.

Dewey's purpose underlying the scientific inquiry in the late nineteenth and early twentieth centuries was to help put an end to the educational approach that served the needs of an elite few. His perception was that the individual student must grow intellectually to become powerful so that he may control his own life, i.e., make decisions for himself.

Many notions of problem solving developed as a follow up of Dewey's work regarding a problem solving approach. There seems to be an agreement among problem solving researchers that the process has several basic phases: a recognition phase, an alternative search phase, an action phase, and an evaluation phase (Kieren et al., 1980). Dewey's strategy is basic to most of the strategies developed with few additional, modified, and/or elaborated steps to meet the needs of a specific subject matter.

Polya: George Polya (1945) presented a four-step comprehensive strategy applicable in the discipline of mathematics which continues to be used widely by mathematicians.

To solve a problem, according to Polya, is:

> to find a way where no way is known off hand, to find a way out of difficulty, to find a way around an obstacle, to attain a desired end, that is not immediately attainable by appropriate means.[2]

Polya, like Dewey, believes that solving problems can be regarded as the most characteristically human activity. Therefore, in his works he aims to understand this activity, he proposes means to teach it, and eventually to improve the problem solving ability of the student.

Polya's strategy is widely known as an "heuristic model". It is simple, well defined, and has four phases:

1. Understand the Problem: In understanding the problem an individual must determine:
   (a) what is unknown?
   (b) what are the data?
   (c) are too many or too few facts given?
   (d) can the facts be restated more simply?
   (e) can a figure be drawn and suitable notation introduced?
2. Devise a Plan: In devising a plan to solve the problem, one must:
   (a) search for a similar or related problem,
   (b) guess a solution. Does the solution solve the problem?
   (c) see if there is a possible pattern of the problem.
   (d) solve part of the problem,
   (e) see if the problem can be solved by adding, subtracting, dividing or multiplying.
   (f) develop a chart, use diagrams or pictures to decide on the solution.
3. Carry Out the Plan: In conducting and carrying out the plan, one must check:
   (a) each step to prove or refute it to reach the

result,

(b) or prove that it is correct?

4. <u>Look Back:</u>

(a) Check the results of the steps involved,

(b) alter the approach to see if you still get the same answer,

(c) keep track of and utilize past solutions set on other problems.

The Polya model is taken as the basic frame of reference to heuristic processes in mathematical problem solving. The four phases described above do not occur in sequence, and all the four phases may not be utilized in solving a problem. The first three phases correspond to activities described above. The fourth phase is somewhat unique and deserves a comment. The looking back phase helps generate an appropriate method or useful result applicable to the new situation, and hence affords opportunities for transfers.

Polya's heuristic model just described forms the frame of reference for this research and gives direction to the questions posed.

<u>LeBlanc</u>: Another variation on the four phase strategy was put forth by John LeBlanc (1977). In most problems, according to LeBlanc, the process of finding a solution can be divided into a number of steps or phases. Each of the steps has a somewhat different focus, but their

common goal is the solution of a problem.

The four stages are:

1. Understanding the Problem: In seeking to understand the problem one may ask, "Is the statement of the problem clear? Are the words and mathematical symbols familiar?"[3] Sketching figures or constructing diagrams or tables may be helpful at this stage. Rereading the statement several times may be helpful after an unsuccessful attempt.

2. Planning To Solve the Problem: Getting started on the plan is the most difficult phase in problem solving. It is at this phase that the problem solving heuristics/strategies are introduced. These may change several times as attempts are being made to carry out the plan.

   For some students complete understanding of the problem may come at this stage. Some problem solving strategies that are helpful to solve the problem are organized listing, guess and check, experimenting, drawing diagrams, tables and graphs, deduction, and searching for a pattern.

   LeBlanc states that "in problem solving an individually acquired set of processes in brought to bear on a situation that confronts the individual."[4]

3. Solving the Problem: The plan or strategy selected in step two is carried out in this phase. This step

should be viewed as the one that carries out the plan. If the plan does not work the problem solver should return to step two to devise another plan. Carrying out the idea is easier than creating it, so this phase of problem solving may be more routine than the others, checking occasionally that whatever is being done is leading towards the situation.

4. Reviewing the Problem and the Solution: There are two aspects to this step: One is looking back over the steps taken, and the other is extending the problem situation to create variations on an entirely new problem. This helps to identify similarities in problems.

All the four steps taken together form the frame for problem solving. There is a top-to-bottom order implied, but one rarely proceeds through the phases exactly once in solving the problem. Some steps are repeated before the problem is solved.

Popp-Robinson Model: A general problem solving strategy recently developed is the Basic Inquiry Model (Research Study Skills, 1979) to be used as a general approach to the problem solving content of the school program. An adaptation of this model is the Mathematics Problem Solving Strategy (Popp, Robinson and Robinson, 1974; Popp and Seim, 1978) which is another good example of mathematics problem solving. It consists of the following steps:

1. Problem: What is the problem? Read and reread the problem content until you grasp it.
2. Question: I.e., identify the essential focus (the unknown), one that is worth pursuing. The student has to know enough of the problem in order to be able to form a reasonable question.
3. Organization: Organization is further broken down into:
   (a) analysis -- identify the essential elements and their relationships,
   (b) information -- identify given information, recall or locate other relevant information, identify applicable relations,
   (c) representation -- representation of the problem is closely related to the nature of the question, so more attention should be paid here than usual.
4. Calculation: Select appropriate algorithms or strategies, sequence steps, calculate.
5. Conclusion: Write the results of the calculations in a precise required manner.
6. Record: Record the conclusion for future reference.
7. Evaluate: (a) determine whether the answer obtained to a mathematics problem match, type of answer with type of question/problem,
   (b) approximate any calculations done to check reasonableness of answer obtained.

The classification chart (see Figure 1) compares at a glance the common points in the problem solving strategies presented by Dewey, Polya, and Popp and Robinson, which have already been described in some detail.

Gestalt psychologists like Karl Duncker and Max Wertheimer do not appear on the chart as they failed to present any specific or general strategy for problem solving.

To sum up Duncker,

> the final form of a solution is typically attained by way of mediating phases of the process, of which each one, in retrospect, possesses the character of a solution, and in prospect, that of a problem.[5]

Wertheimer emphasized the integration of past experiences in problem solving, but never offered a satisfactory explanation of the nature of past experiences. Wertheimer wrote that generally speaking there is first a situation, S_1, the situation in which the actual thought process starts, and then after a number of steps, S_2, in which the process ends, the problem is solved (Smith, 1973). What happens between S_1 and S_2?

The Gestaltists emphasize the sudden reorganization of experience, the insightful moment, but provide little additional help.

From among all the strategies discussed, Polya suggests many general heuristics which are clear, precise,

DEWEY	POLYA	POPP & ROBINSON
1. Problem is felt.	1. Understand the problem.	1. Investigate the context to establish the general nature of the problem.
2. Problem is located and defined.		2. Identify the essential focus.
		3. Organisation: (a) Analysis (b) Information (c) Representation.
3. Possible solutions suggested.	2. Devise a plan to solve the problem.	4. Calculation.
4. Reasoning of the solutions suggested.		
	3. Carry out the plan.	5. Conclusion.
		6. Record.
5. Evaluation of the solution.	4. Look back and examine the solution.	7. Evaluation.

Figure 1

Classification Chart for Problem Solving Models comprehensive, and are widely used today in mathematics textbooks at all grade levels. Some of the heuristics he presents are: what is unknown? what are the data? draw a figure, chart or diagram; search for a similar or related problem; guess a solution; check the solution by looking back; is there a possible pattern of the problem? can the problem be solved by adding, subtracting, dividing or multiplying? All of these heuristics have gained attention today because of their successful incorporation into the problem solving programs in mathematics textbooks. They are taught for the simple reason that they seem to be central to the problem solving process.

These general heuristics of Polya, adopted by many mathematics educators and textbook writers, examples of which are discussed later, form the basis of the problem solving strategies that the investigator has used to analyze the exercises and problems of grade three mathematics textbooks.

Kinds of Problems That Appear in Mathematics Textbooks

Frequently problems in mathematics are classified into two types:

1. Textbook Problems: A textbook problem introduces or follows the development of an arithmetic operation. It can be solved by direct application of one or morepreviously learned algorithms, i.e., step-by-step procedures followed in a strict order, at the same time identifying the operation that is appropriate for solving the problem. The objective of these problems is to strengthen the recall of basic facts, operations, algorithms and reinforce the relationship between the operations and their application in day-to-day life situations.

    A standard textbook problem requires the child to translate a real world situation into mathematical symbols so that a previously learned algorithm can be used to solve the problem. The problem situation is normally presented using pictures, short phrases or sentences, paragraphs, or a combination of these models. In the first and second grades, pictures alone or pictures and words ("rebus format") are commonly used to present the situation. In the middle grades fewer pictures are used and abbreviated story problems often occur.

The number of steps is another factor in the difficulty level of the problem. A two-step problem is more difficult than a one-step problem providing the other factors related remain constant. In one-step problems, when concepts and relationships involved are understood, the child has only to choose the appropriate arithmetic operation (addition, subtraction, multiplication, division) to solve it.

In multi-step arithmetic problems, besides choosing the operations to apply, one has to plan and organize the order of applying the operations, and decide to which pair of numbers should each pair be applied. Multi-step problems are prevalent in upper grade levels, but they could be incorporated into lower grades providing they are appropriate for the maturity level of the students.

A few examples of textbook problems follow.

1. A sporting goods store has 247 baseballs worth \$2.37 each and 142 footballs worth \$3.84 each. What is the total cost of all of the baseballs and footballs?
2. Ninety-six children are to be placed in rows, with eight children in each row. How many rows will there be?
3. Mary and Sally go to the store. Mary has 58¢ and Sally has 62¢. They want to buy a Frisbee that

costs 92¢. How much change will they get back?

These textbook problems ask that the child know the algorithm as well as the operation to solve the problem. Textbook problems can also be presented with a minimum of situational information, for example:

1. 16 children were in the hall
   4 children went to classroom
   6 children went in the playground.
   How many children were left in the hall?
2. Sheila had 70¢.
   She spent 45¢.
   She earned another 30¢.
   How much does Sheila have now?
3. 3 cartons
   6 bottles in each carton
   How many bottles in all?

These problems are sometimes called "telegraphic style" problems.

2. <u>Process Problems</u>: Process problems require the use of strategies or some non-algorithmic often heuristic approach for a solution. This type of problem stresses the process of obtaining the solution rather than the solution itself, using one or more strategies and sometimes having more than one answer. Process problems are used to encourage the development

and practice of problem solving strategies.

In the first part of this chapter a number of strategies promoted by different theorists are mentioned. Since this practice in using strategies also helps in the thinking process, it is expected that mathematics textbooks and teachers would put a high value on process problems.

3. <u>Problem-Solving Staff in Mathematics</u>: The problem solving staff in mathematics (1977) of the Oregon Department of Education, quoted by Richard Brannon, and Oscar Schaaf, suggested a set of problem solving strategies that should be integrated into the regular curriculum, to be part of everyday mathematics classes. Note that these are based on Polya's heuristics.

1. Guess and Check: Each row, column, and diagonal of this magic square adds to 15. Use the numbers 2, 3, 5, 6, 8, 9 to complete the magic square.

2	9	4
7	5	3
6	1	8

This example can also be solved by a "trial and error" strategy.

2. Look for a Pattern:

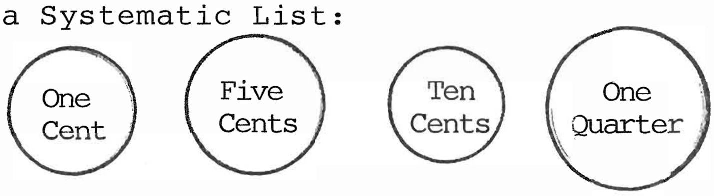
$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$
$$1234 \times 8 + 4 = 9876$$

(i) Predict the answer for 123,456 X 8 + 6 =

(ii) Check your prediction.

(iii) Predict and check 12,345,678 X 8 + 8 =

3. Make a Systematic List:



The coins shown above are the only ones you have. What amounts can you make if you use 1, 2, 3, or 4 of the coins?

4. Make a Drawing:

A fireman stood on the middle step of a ladder. As the smoke got less, he climbed up three steps. The fire got worse so he had to climb down five steps. Then he climbed up the last six steps and was at the top of the ladder.

How many steps were in the ladder?

5. Make a Reasonable Estimate:

Megan is at a material shop. She has a $10.00 bill. Which of these purchases can she make?

(a) 3 metres of material at $2.98 a metre.

(b) A pair of scissors for $8.15 plus a spool of thread at 85¢.

(c) A pattern for $2.75 plus 4 metres of material at $2.10 a metre.

(d) Four sets of buttons at $1.25 a set and 2 balls of yarn at $1.95 a ball.

6. Eliminate Possibilities:

Ms. Ashley has less than 100 pieces of candy.

If she makes groups of 2 pieces, she will have 1 piece left over.

If she makes groups of 3 pieces, she will have 1 piece left over.

If she makes groups of 4 pieces, she will have 1 piece left over.

If she makes groups of 5 pieces, she will have no pieces left over.

How many pieces of candy could she have?

4. <u>The Mathematics-Methods Program</u>:

The Mathematics-Methods Program developed by the Indiana University Mathematics Education Development Centre during the years 1971-75, to which John LeBlanc was one of the substantial contributors, states that one of the goals of this program is the discovery of strategies for solving real world concerns. Again, these strategies are based on Polya's heuristics. They suggest that problems used should highlight the steps in the problem solving process which tend to hold high interest for the students. Problem solving strategies with examples follow:

1. Draw a Sketch, Diagram, Table:

(a) Given "n" points in a plane, no three of which are in a straight line. How many line segments must be drawn to connect all pairs of points?

(b) How many diagonals are there in a regular n-sided polygon?

(c) Fifteen couples have been invited to a birthday party. The host has several small card tables that can seat one person on a side. He plans to set the small tables end to end to make one long table to seat all the guests. How many of the small tables will be needed to seat the fifteen couples?

2. Look for a Pattern:

(a) Find at least five number patterns in the array.

Hint: Given array as a portion of the larger array.

1 2 3 5 8 (5 = 2 + 3
8 = 5 + 3)

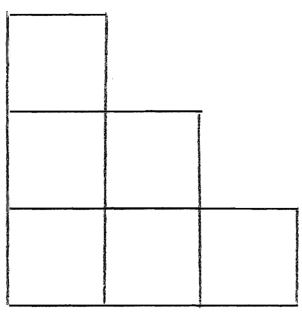
$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

(b) Continue the array for two more rows using one or more of the patterns you found in (a).

(c) Continue the array shown below for two more rows using a number pattern you find in the displayed rows.

				1				
			1		1			
		1		3		1		
	1		5		5		1	
1		7		13		7		1

(d) Six blocks are used in a staircase that has 3 steps.



How many blocks are needed for a staircase that has 4 steps?

Can you make a staircase with 28 blocks?

With 34 blocks?

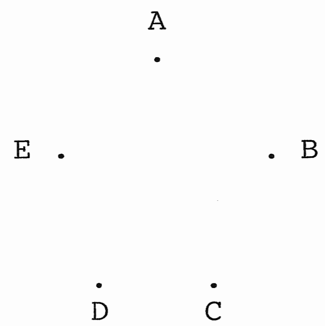
Hint: For every single step added you have to add blocks equal to the total number of steps.

E.g., for 4 steps add 4 blocks

for 5 steps add 5 blocks.

3. Guess and Check:

(a) Connect the dots.



There are five dots arranged in a pentagon. In how many different ways can four or fewer straight line segments connecting the dots be drawn?

Hint: It is the connections of the dots that are important not the way the figure is drawn.

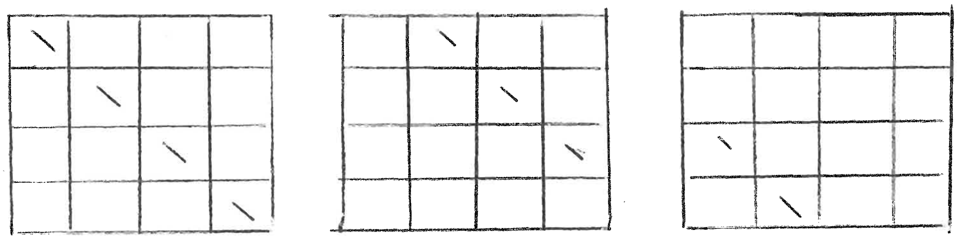
(b) Placing pennies.

Place four pennies on a 4 x 4 grid in such a way that no two are in the same row, column, or diagnal.

Hint: Two pennies are in the same row if they are in the same horizontal line.

They are in the same column if they are in the same vertical line.

Three of the five upper left to lower right diagonals are shown by the dashed lines below. There are also five upper right to lower left diagonals.



4. Make a Systematic List:

(a) Several of the twelve teachers in the local elementary school plan to try to grow vegetables in their science classes. If six teachers plan to grow beans, eight plan to grow corn, and three plan to grow both, how many plan to grow neither beans nor corn?

(b) A family of four -- two parents, a son and a daughter -- have a set of Christmas cups. One evening they observed that mother had the cup decorated with candles, father had the one decorated with holly, the son had the one decorated with carolers, and the daughter had the one decorated with angels. In how many ways can the cups be distributed the next evening so that no individual has the same cup?

(c) Three missionaries and three cannibals wish to cross a river. There is a boat which can carry three people, and either a missionary or a cannibal can operate the boat.

It is never permissible for cannibals to outnumber missionaries, neither in the boat nor on either shore.

What is the smallest number of trips necessary to make the crossing?

How many trips are necessary if the boat holds only two people?

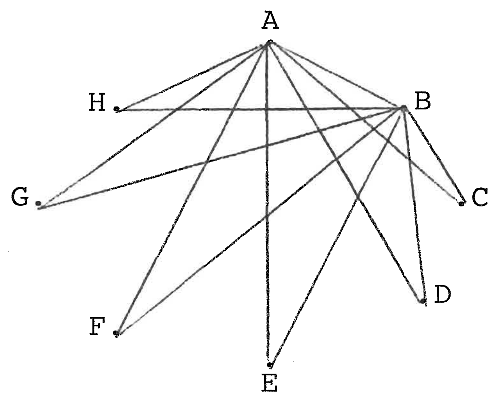
(d) When stamps are purchased at the post office, they are usually attached to each other. In how many ways can five stamps be attached to each other?

In teaching problem solving, (Leblanc, 1977), the instructional goal should be to help children learn procedures for solving problems. An individually acquired set of processes is brought to bear on a situation that confronts the individual. A procedure or strategy may be thought of as an overall plan designed to solve the problem. Certain problems can be solved using a variety of problem solving strategies. For example,

1. Make a Drawing:

There are eight people at a party. If each person shakes hands with other guests, how many handshakes will there be?

Hint: Draw eight dots in a circle using lines to show handshakes.



2. Act It Out:

Have eight children shake hands with each other counting as this is done.

3. Make an Organized Listing:

List names of eight children each shaking hands with the other.

Jim	Jane	John	Joan	Bob	Beth	Bill	Barb
Jane	John	--	--	-	--	--	
John	Joan	--	--	-	--		
Joan	Bob	--	--	--			
--	--	--	--				
--	--	--					
--	--						
--							

McKillip and Davis (1980) believe that teaching problem solving is the major vehicle, sometimes the only one, through which we address applications of mathematics. Success in problem solving is enhanced by:

(a) knowledge of the given conditions at hand, to better understand the problem, and

(b) knowledge of problem solving processes to solve the problem.

They suggest three strategies dealing with process problems.

1. Representing a problem by a <u>sketch or drawing</u> helps the student to understand a problem's applicability to real life, e.g.,

   (a) John, Alex, and Maryann live on the same road. John lives 10 kms. from Alex. Maryann lives 2 kms. from Alex. How far does John live from Maryann?

2. Although not frequently used as a problem solving procedure in elementary school, trial and error is a good problem solving procedure, especially when calculators are available. This strategy places many difficult problems in the range of elementary students, e.g.,

   (a) I am thinking of two 2 digit numbers. They have the same digits. The sum of the digits of each number is 10 and the difference between the number is 18. What are the numbers?

(b) Jennifer's dad pays $28.00 a month for her ballet lessons. If she goes to ballet on Monday and Thursday each week, about how much does each lesson cost?

3. Looking Back by way of reviewing the procedures used in solving a problem reinforces such techniques as careful reading, drawing pictures, organizing guesses, and reading carefully so as to be sure of what is known and what we need to find. These are the very things we want children to learn when they need problems solved or any situation in which mathematics must be applied.

In choosing process problems for problem solving instruction, LeBlanc, Proudfit and Putt (1980) advise that care be taken to choose problems that lend themselves to solution using a variety of strategies. These strategies provide an opportunity for students to devise creative methods of solution and allow them to enjoy mathematical problem solving. These are some of the problem solving strategies (with examples) that they offer:

1. Guess and Check:

(a) Jesse's mother paid him $1.60 allowance in quarters, dimes, and nickles. He received 17 coins in all. How many of each coin could his mother have given him?

(b) Jane saw 18 chickens and pigs in a farmyard. If she counted fifty legs, how many chickens and how many pigs were in the farmyard?

2. Make a Table/List:

Susan wanted to buy a candy bar that cost 25¢. The machine would take pennies, nickles, and dimes in any combination. List the different coins she could use to pay for her candy bar.

This problem can also be solved by strategy (1).

Example (ii) in strategy (a) can also be solved using this strategy.

3. Working Backwards:

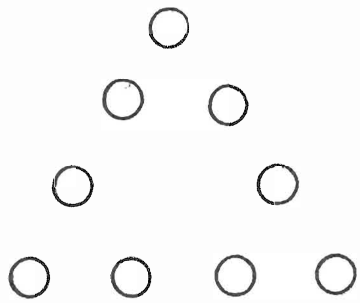
Working backwards on a problem already solved is essential for consolidating the knowledge gained from the situation and for developing in children the process needed for solving problems. Therefore, it should not be omitted from the instructional sequence. A number of different strategies for each problem should be shared, appropriately labelled and emphasized.

In the past arithmetic has dominated much of the early school mathematics curriculum. The emphasis has been on learning and computational algorithms. Musser and Shaughnessy (1980) think the emphasis should be on developing and using strategies to solve problems. They suggest three problem solving strategies, examples of which follow .

1. Trial and Error:

Trial and error simply involves applying allowable operations to the information given.

(a) Arrange the digits 1-9 in the triangle shown so that the sum of each side is 17.

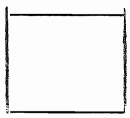


(b) Is there another "side sum" (other than 17) using the digits 1-9? What are all the side sums?

2. Patterns:

The pattern strategy looks at selected instances of the problem. Then a solution is found by generalizing from the specific solutions.

(a) Given 1 x 1 square of toothpicks, how many toothpicks are needed to build the 1 x 1 square into a 2 x 2 square or 4 x 4 square?



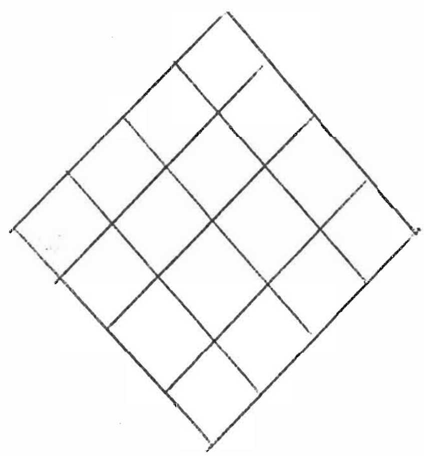
1 x 1 square

3. Solving a Simpler Problem:

This strategy may involve solving a "special case" of a problem or temporarily retreating from a complicated problem to a shortened version. The simpler problem strategy is often accompanied by pattern strategy.

(a) How many different downward paths from A to B are in the grid in the following figure?

A



B

(b) How many squares in an 8 x 8 checkerboard?

Solution hints:

Complete this table and notice a pattern. Both strategies "solve a simpler problem" and "patterns" are used to solve the problem.

SIZE OF SQUARES

		1 x 1	2 x 2	3 x 3	-----	8 x 8
	1 x 1	1				
	2 x 2	4				
	3 x 3	9	4	1		
Size of	-					
largest	-					
square	8 x 8					

Strategies Related to Different Studies

Reviewing the studies mentioned, namely those of PSM staff (1977); LeBlanc (1971-75); LeBlanc et al. (1977); McKillip and Davis (1980); Musser and Shaughnessy (1980); it was found that:

Nearly all of the investigators suggested the following problem solving strategies in mathematical process problems.

(1) Guess and check.

(ii) Make a systematic list.

(iii) Make a drawing, chart, table.

(iv) Experimenting the problem or acting it out.

(v) Look for a pattern.

(vi) Estimate.

Trial and error was suggested by LeBlanc (1977), McKillip and Davis (1980), and Musser and Shaughnessy (1980).

Working backwards was suggested by LeBlanc, Proudfit, and Putt (1980); McKillip and Davis (1980).

It was also observed that all of the strategies used by these investigators have a common base in Polya's model (p. 15) for problem solving. This survey supports the selection of Polya as the framework for this study. From the survey and Polya's work is derived the following list of strategies that will be used to build an analysis chart and pose a set of questions for examining grade three mathematics textbooks.

(i) Guess and check.

(ii) Make a drawing, chart, table.

(iii) Look for a pattern.

(iv) Make a systematic list.

(v) Act out the problem.

(vi) Eliminate possibilities.

Questions Posed By the Study

With the two main types of mathematics problems identified as textbook and process, and with a list of process strategies determined, the specific questions to be asked by the study can now be framed.

1. What is the frequency of occurrence of "one-step" problems contained in the exercises and problems of grade three mathematics textbooks?
2. What is the frequency of occurrence of "two-steps" problems contained in the exercises and problems of grade three mathematics textbooks?
3. What is the frequency of occurrence of "multi-step" problems contained in the exercises and problems of grade three mathematics textbooks?
4. What is the frequency of occurrence of the "guess and check" strategy contained in the exercises and problems contained in grade three mathematics textbooks?

5. What is the frequency of occurrence of "look for a pattern" strategy contained in the exercises and problems of grade three mathematics textbooks?
6. What is the frequency of occurrence of the "make a drawing" strategy contained in the exercises and problems of grade three mathematics textbooks?
7. What is the frequency of occurrence of the "make a systematic list" strategy contained in the exercises and problems of grade three mathematics textbooks?
8. What is the frequency of occurrence of the "eliminate possibilities" strategy contained in the exercises and problems of grade three mathematics textbooks?
9. What is the frequency of occurrence of the "act it out" strategy contained in the exercises and problems of grade three mathematics textbooks?

Summary

Chapter Two begins with an elaboration of the notion of problem solving as it has been developed in the educational and psychological literature. This is followed by the kinds of problems that commonly appear in mathematics textbooks. The chapter ends by looking at the kinds of strategies used for problem solving in different mathematics programs, and poses nine specific questions to be answered by the study.

Footnotes - Chapter Two

1. Dewey, John. The Theory of Inquiry in Logic, New York: Holt, Rinehart and Winston, 1938, p. 17.

2. Polya, G. "On solving mathematical problems in high school,"; Krulik, S., and Reys, R.E. (eds.) "Problem solving in school mathematics," National Council of Teachers of Mathematics, Reston, Virginia, 1980, p. 1.

3. LeBlanc, John F.; Kerr, Donald R.; Thompson, Maynard; Experiences in Problem Solving, Don Mills, Ontario: Addison-Wesley Publishing Company, 1975, p. 6.

4. LeBlanc, John F. "You can teach problem solving," Arithmetic Teacher, V25, N2, Nov. 1977, p. 16.

5. Duncker, Karl. "On problem solving." Psychological Monographs, No. 270, American Psychological Association, Washington, D.C., 1945, p. 7.

CHAPTER THREE

RELATED RESEARCH

In reviewing current research in mathematics education, Romberg (1969) reported little work especially concerned with mathematics texts. He categorized mathematics research in the following eight areas:

(1) mathematical learning from an association learning framework;

(2) mathematical learning from an activity learning framework;

(3) mathematical problem solving and creative behaviour;

(4) mathematics teaching;

(5) the effectiveness of instructional programs;

(6) association of learner characteristics with mathematical achievement;

(7) attitudes towards mathematics;

(8) the evaluation and measurement of mathematics achievement.

Most of the studies that he reported were small one shot affairs in which no significant differences were found (Kane and Holz, 1972). The main emphasis of these studies appeared to focus on the mathematical content, i. e. , could a certain topic be taught to a given group of students, and also on the evaluation of pupil achievement.

A monumental survey by Suydam (1976) of published research on elementary school mathematics from 1900 to 1965 yielded 84 studies of problem solving. Generally these studies were of low quality with conflicting results (Kilpatrick, 1969).

A study of the characteristics of elementary school mathematics programs in Ontario (Russell, Robinson, Wolfe and Diamond, 1975) was conducted on a random stratified sample of eighty-five schools with 2% population across Ontario. The data in this research was collected by administering questionnaires to the principals and teachers of the sample schools (grades 1, 3, 5 and 8) to assess the effectiveness of elementary mathematics programs in their schools. This was accompanied by the observation of classroom procedures during the mathematics period.

One of the many conclusions the investigators came up with was that there is much emphasis on computation while students' problem solving skills are unsatisfactory. Teachers and principals of the sample schools expressed most difficulty with topics on problem solving. The researchers further noted that the resource material was present in words only and that whatever was available was infrequently found in the classrooms and was underutilized.

Word Problems

Gorman (1967), in a systematic and critical analysis, identified 293 studies on word problems conducted between 1925 and 1965. Only 37 of these studies, mostly doctoral theses, were deemed "acceptable" (Kilpatrick, 1969) according to Gorman's criterion of high internal validity.

A later survey done by Marilyn S. Suydam (1980), suggests that most of the problem solving research done in this century deals with word story problems. Since word problems form part of this research, a few selected studies that seemed most relevant to mathematics are mentioned here.

Children at the elementary level are better able to solve word problems in a drawn format than in a telegraphic form (Sowder, Moyer, Sowder, 1984).

Third grade children transform the mathematical presentation of the comparison-type word problems to a result by following a step-by-step strategy from the rich or poor repertoire they possess (Mary, 1985). Furthermore, third graders appear to invent many new strategies or heuristics in order to adapt the taught algorithms to their own understanding and to specific word problems confronting them.

When primary grade children were taught to use specific problem solving strategies in solving arithmetic story problems, the instruction resulted in significant improvement on:

(i) finding the answer,

(ii) drawing the diagram, and

(iii) writing the appropriate number sentence (Lindvall and others, 1982).

Heuristic solutions involving awareness of the method may result in greater long-range efficiency than conventional solutions (Lucas, 1972).

Research on one-step multiplication problems (Quintero, 1980) and on one-step addition and subtraction word problems has shown that the concepts and relationships involved in the problem are strong determiners of problem difficulty (Quintero, 1983).

The results of the last two British Columbia Mathematics Assessments (1977, 1981) demonstrated that a drastic difference in performance existed among elementary school students with respect to solving multi-step mathematical word problems as opposed to one-step problems.

Subjects who were taught strategies for solving mathematics word problems were more successful in applying these strategies than subjects that were not taught (Cloer, 1981). Moreover, a knowledge of the number system, understanding of arithmetic concepts and relationships, and computation ability influenced a subject's success in mathematics word problems.

In solving one-step word problems, the primary grade children's difficulty was due to the lack of knowledge of what operation to use (Knifong and Halton, 1977).

> It is worth noting that several current textbooks have outlined a series of problem solving steps for students. One of these steps involves deciding what operation is appropriate. However, little, if any, guidance is given to students in helping them learn how to make this decision.[1]

In order to achieve this, children can be asked to manipulate objects, act out a situation, draw a picture to show the problem. They can be asked to match one of several models to a problem, create problems to match a model. In each case the emphasis should be on modelling and not on getting an answer. This is what Polya's philosophy and model are about.

Mathematics Textbooks

Research which effectively studies printed mathematics materials is indeed sparse. Other than in the areas of sequence theory and readability there appears to be almost no research designed to systematically study mathematics texts (Holz and Kane, 1972). This conclusion by Holz and Kane help to explain the difficulty in finding studies directed to this investigator's topic.

Al-Saloom (1981) selected mathematics textbooks, grades four through six in order to examine, through content analysis, the frequency of occurrence of cognitive processes contained in the problems and exercises of these texts. He chose a sample of twenty-five percent of the problems and exercises in the texts, and used Romberg's (1969) seven levels of cognitive behaviour to classify the sample. Romberg's seven levels are knowing, translating, manipulating, applying, analyzing, synthesizing and evaluating. Each textbook was rated by two coders. Consistency over time (pre-analysis and post-analysis), and consistency among analyses was maintained by the researcher. He used the chi-square method to analyze the data.

The major findings of this research were that emphasis was placed upon low level cognitive behaviour in all three grades. Fourth grade mathematics textbooks had more emphasis on high level cognitive behaviours than did fifth or sixth grade textbooks.

Al-Saloom (1981) mentioned research done by Passi (1970), Cruickshank (1968), Dickinson and Russell (1971) because of their relevant method of content analysis. Where Passi and Cruickshank studied mathematics content, Dickinson and Russell studied the literature of journals in adult education. Beyond the method used (i.e., content analysis), the investigator did not deem the findings of these studies particularly insightful for the present work.

Kane and Holz (1972) developed a technique for studying the organization of mathematics text materials, grades four to twelve. This technique is called the C.M.M.T. (Classifying Messages in Mathematics Texts). It is used to identify and study presentation variables in mathematics texts. There are two dimensions to the technique, Content mode, and Representation mode. Exercises and problems that appear in textbooks are classified into categories: nine categories in the Content mode and seven categories in the Representation mode. Each category is described in terms of skill and is given a rank number. There are nine ranks (skills) in the Content category and seven ranks (skills) in the Representation mode.

<u>Categories of the Content Mode</u>

1) Definition - meaning of words and symbols.
2) Generalization - imparting rules, axioms, theorems, formulas, etc.
3) Specific explanation - concrete examples, discussion, etc.
4) General explanation - proof of propositions, general discussion, etc.
5) Procedural instruction - directions.
6) Developing content - questions in exposition, developmental activities, guided discovery, exercises, etc.
7) Understanding developed content - real world problems, applications of generalizations in concrete situations.
9) Analyzing and synthesizing developed content - proving proposition, finding new relationship, unguided discovery.

<u>Categories of the Representation Mode</u>

1) Words.

2) Mathematics symbols.

3) Representation of abstract ideas - Venn diagrams, geometric diagrams, mapping pictures.

4) Graphs - number lines, coordinate graphs, bar graphs, etc.

5) Representation of physical objects of situations - plans, maps, cross-sectional drawings, photographs, etc.

6) Non-mathematical illustration - motivational photographs, cartoons, etc.

7) Combination of illustration with written text - flowcharts, mathematics tables, tree diagrams.

In classifying textual material the category rank number in each of the two dimensions, Content mode and Representation mode, is recorded in sequence following the natural flow of the printed material.

Quantitative aspects are described by determining the proportions of messages of various types and logical combinations of types. The basic unit of measure used is one-fourth of a line of print. Each passage in each textbook is analysed by two raters. Inter-relater reliability is adapted from interaction analysis.

Results of the C.M.M.T. technique showed that the most numerous categories of the mathematics text materials investigated were specific explanation (Rank No. 3), general explanation (Rank No. 4), and understanding a developed content (Rank No. 7), all of the Content mode. The other categories tended to appear relatively infrequently. Mutual exclusiveness appears as a possible problem in the exercise and problem categories on dimension one - i.e., the Content mode, for example, real world problems can be of a rote or practice and application (Rank No. 8). The researchers recommended that by putting more exercises in a unit, more categories than general explanation could be used. They also noted that illustrations tended to take up large areas of a mathematics text. The C.M.M.T. technique was found to be extremely time consuming. However, it gives another approach to content analysis by determining the proportions of various types of messages and their combinations. Regarding problem solving, the researchers noted that more space in the text was taken by illustrations as compared to exercises and problems.

Given the paucity of closely related studies, this research must be seen as largely the work of a single investigator. She hopes that this work may be useful in establishing a small link in the much needed effort among mathematics educators to carry on studies like the one being reported.

Summary

This chapter includes a review of the literature related to problem solving in general, and word problems in particular, and gives details of one research study found to be most closely related to the topic of this study.

Footnotes - Chapter Three

1. Rathmell, Edward C. <u>Teaching Children to Solve One-Step Word Problems</u>, ED 207849, 1981, p. 4.

CHAPTER FOUR

THE RESEARCH PROCEDURE

Sample

From a total of six grade three mathematics textbooks listed in Circular 14 Textbooks 1984, three were selected arbitrarily (every other) for this study. These three are listed below.

1. Bornhold, Donald L.; Gutcher, Robert; Lindermere, Linda; Tossell, Stella; Traynor, Cathie. Starting Points In Mathematics 3, Ginn and Company, 1977. (Textbook I). This textbook has sixteen units.
2. Bates, John H.; Clifford, Terry J. Math 3, McGraw-Hill Ryerson Limited, 1982. (Textbook II). This textbook has nine units.
3. Super, Doug; Carlson, Florine Koko; Burbank, Irvin K. Houghton Mifflin Mathematics 3. Houghton Mifflin Canada Limited, 1981. (Textbook III). This textbook has fifteen units.

These three grade three mathematics textbooks will be referred to as Textbook I, Textbook II and Textbook III in this report.

Tables 1, 2, and 3 present the total content of

Table 1

Content and Sub-Sample Units - Textbook I

Unit	Page	Content	Units Selected	Activities in Units* Selected	Activities in all Units
1	2- 25	Numeration	✓	321	321
2	26- 47	Addition and Subtraction Facts	✓	393	393
3	48- 67	Addition			253
4	68- 79	Geometry	✓	150	150
5	80- 99	Subtraction			276
6	100-121	Measurement			93
7	122-139	Decimals	✓	255	255
8	140-161	Multiplication	✓	284	284
9	162-179	Exploring Division	✓	204	204
10	180-201	Geometry and Measurement	✓	313	313
11	202-213	Numeration			214
12	214-233	Addition	✓	203	203
13	234-251	Subtraction	✓	221	221
14	252-267	Decimals			363
15	268-291	Multiplication			418
16	292-312	Division	✓	441	441
			TOTAL	2,785	4,042

*Activities refer to problems and exercises having problem solving plus problems and exercises not having problem solving.

TABLE 2

Content and Sub-Sample Units - Textbook II

Unit	Page	Content	Units Selected	Activities in Units Selected	Activities in all Units
1	1- 28	Number Review	✓	474	474
2	29- 60	Addition and Subtraction			481
3	61- 92	Addition and Subtraction. Measurement			352
4	93-124	Addition and Subtraction. Graphing	✓	393	393
5	125-156	Multiplication. Geometry	✓	262	262
6	157-188	Division. Measurement	✓	342	342
7	189-220	Fractions and Decimals. Geometry	✓	248	248
8	221-252	Multiplication. Measurement	✓	428	428
9	253-282	Division. Geometry	✓	393	393
			TOTAL	2,540	3,373

TABLE 3

Content and Sub-Sample Units - Textbook III

Unit	Page	Content	Units Selected	Activities in Units Selected	Activities in all Units
1	1- 19	Addition Facts	✓	324	324
2	20- 39	Subtraction Facts	✓	377	377
3	40- 59	Numerals to 9999	✓	351	351
4	60- 79	Addition I	✓	408	408
5	80- 99	Subtraction I			411
6	100-119	Measurement	✓	309	309
7	120-139	Multiplication Facts I			449
8	140-159	Division Facts I	✓	354	354
9	160-179	Addition II			444
10	180-199	Subtraction II			408
11	200-219	Geometry and Graphs	✓	262	262
12	220-239	Multiplication Facts II	✓	269	269
13	240-259	Division Facts II	✓	305	305
14	260-279	Decimals			302
15	280-299	Multiplication	✓	280	280
			TOTAL	3,239	5,253

the three textbooks and identify sub-sample units. They also indicate the total number of activities contained in the whole textbook, and specifically in the units selected for study.

Sub-Sample

For the analysis, an average of nine units were selected from each of the three sample textbooks. These units constitute the sub-sample of this study and represent mathematics curriculum in all the three major areas, i.e., arithmetic, measurement and geometry as they are listed in the Ministry guideline The Formative Years (1975).

Instrument

A checklist was developed for the purpose of analyzing the sub-sample units according to the two dimensions, textbook problems and process problems. Within each dimension there is a finer classification of the difficulty level for textbook problems and problem solving strategies for process problems. Three difficulty levels for textbook problems are identified as one-step, two-step and multi-step, according to the number of operations that are used to solve a problem. In the dimension of process problems there are six problem solving strategies: namely; guess and check, make a drawing, look for a pattern, eliminate possibilities, act it out, and make a systematic list.

One column space in the checklist, labelled "any other" was allocated to allow for strategies used to solve a problem or exercise, but not listed in the checklist. This space was also used to identify any feature of the textbook that the researcher or the two coders thought needed a comment. For a copy of the checklist see Appendix A.

Validity

Validity of the checklist was established by expert opinion sought from mathematics supervisors from three different Boards of Education in Ontario. Using the checklist, a random thirty percent of the sub-sample units were analyzed by the researcher. A booklet explaining in detail the dimensions of textbook and process problems, and giving examples, was given as a guide to the three supervisors. Their suggestions on the placement of a problem or exercise in a certain category, seen differently by the researcher before pre-analysis, were incorporated in revisions to refine the analysis.

Data Collection

Content analysis was used for collecting and categorizing data by two trained coders and the researcher using the checklist (Appendix A). If an exercise or problem was analyzed as a textbook problem, it was checkmarked under the appropriate difficulty level column according to the number of operations used to solve it. If an exercise or a problem was analyzed as a process problem it was checkmarked under the appropriate problem solving strategy that would solve the problem. If a process problem or exercise could be solved by two or more strategies it was checkmarked under all of the appropriate strategies and counted as two or more problems according to the number of problem solving strategies it could be solved with.

To facilitate the process, a booklet defining textbook problems and process problems giving examples was given to the coders.

Content Analysis:

Quantitative content analysis is a research technique for obtaining descriptive data on content variables.

> It offers the possibility of obtaining more precise, objective and reliable observations about the frequency with which given content characteristics occur singly or in conjunction with one another.[1]

Reliability:

Reliability in content analysis means how well the coders recognize the instructions that are given by the researcher. It has two requirements.

1. Objectivity:

   The method or technique that is used to obtain data has to be very precise and clear so that the coders apply it in exactly the same way.

2. Systematisation:

   To ascertain this requirement, analysis should be done through established categories. Content that is to be selected from the whole text should fit the analyst's thesis using Polya's heuristic model. Analyzing problems according to the textbook and process problems, fulfills this requirement.

Another factor of prime importance in content analysis is consistency. Consistency is based on two procedures.

1. Consistency through time, meaning that a coder or a group of coders should produce the same results when they apply the same set of categories to the same content but at different times.

2. Consistency among analysts, that is, different coders should produce the same results when they apply the same set of categories to the same content.

   To attain internal consistency, two coders were trained for sixteen hours in the use of the analysis procedure, according to the difficulty levels of textbook problems and problem solving strategies, using the checklist. The two coders were given practice in the analysis procedure. Then they analyzed the same thirty percent of the sub-sample units that was analyzed by the researcher for pre-analysis and post-analysis tests.

Following are the results of the tests for consistency.

1. Consistency through time. The investigator found a .78 consistency value for the results of pre-analysis and post-analysis on some units (see Appendices B and C). This consistency value was computed using Pearson Product Moment Correlation Coefficient (Siegel, 1956, p. 203).

$$r = \frac{xy}{\sqrt{x^2 \quad y^2}}$$

where $x = x - \bar{x}$, $\bar{x}$ is the mean of the scores on pre-analysis test,

$y = y - \bar{y}$, $\bar{y}$ is the mean of the scores on post-analysis test,

r = result of correlation coefficient.

2. In measuring the level of agreement (consistency among analysts) between the researcher and the first coder (see Appendices C and D), the researcher found

.78 consistency value between the researcher and the first coder on some units, lower on some and higher on others. This value was computed using Pearson Product Moment Correlation Coefficient.

3. In measuring the level of agreement (consistency among analysts) between the researcher and the second coder (see Appendices C and E), the researcher found .77 consistency value using Pearson Product Moment Correlation Coefficient.

4. In measuring the level of agreement between the two coders (see Appendices D and E), the researcher found a consistency value of .78.

In doing correlations of nine units the lowest correlation in two of the units came out to be .44. The highest correlation among the coders came out to be .99 in one unit.

Six of the nine units had a consistency value of .78. The range of all the units correlated fall between .44 - .99, with most of the units falling at .78. This is considered fair.

Compiling and Tabulating:

The analyzed problems and exercises from each subsample unit of grade three mathematics textbooks were compiled by the frequency count and tabulated according to three difficulty levels in textbook problems and six problem solving strategies in process problems, based on Polya's heuristic model (see Chapter Two).

Comparing Frequencies:

Compiled frequencies of textbook problems and process problems were compared textbook by textbook and topic by topic. Since there are no criteria available for deciding low or high frequency of occurrence of problems, direct results were reported giving frequency count and percentage.

Summary

Chapter Four describes the methodological aspect of the study. It includes the sample, giving its source. It also covers the criterion for selecting of sub-sample units from sample textbooks explaining the data collection procedure. The checklist, which is the instrument for the collection and classification of data, is explained; so is the process for establishing its validity. Consistency values pointing to a fair reliability among three coders is presented. This chapter also covers the compiling and tabulating of the identified frequencies of textbook and process problems, comparing them textbook by textbook and topic by topic.

Footnotes - Chapter Four

1. George, Alexander L. "Quantitative and qualitative approaches to content analysis", Pool de Sola (ed.), Trends in Content Analysis, Urban, University of Illinois Press, 1959, p. 8.

CHAPTER FIVE

ANALYSIS OF DATA AND CONCLUSIONS

This chapter includes a description of the data derived from analyzing the problems and exercises of sub-sample units of three grade three mathematics textbooks, using Polya's heuristic model. Questions posed by this study (Chapter Two) are reviewed here. Answers to these questions are provided based on the analyzed data.

Re-Statement of the Problem

Grade three mathematics textbooks are examined through content analysis in order to determine the extent to which problem solving strategies appear and how well they carry through the Ministry's educational goals of problem solving as set out in The Formative Years (1975).

Regarding the congruence between problem solving strategies and the important place given to the educational objectives of developing problem solving skills, this researcher has tried to answer the following questions:

1. What is the frequency of occurrence of "one-step"

textbook problems contained in grade three mathematics textbooks?

2. What is the frequency of occurrence of "two-step" textbook problems contained in grade three mathematics textbooks?
3. What is the frequency of occurrence of "multi-step" textbook problems contained in grade three mathematics textbooks?
4. What is the frequency of occurrence of the "guess and check" strategy contained in grade three mathematics textbooks?
5. What is the frequency of occurrence of the "make a drawing, chart, table" strategy contained in grade three mathematics textbooks?
6. What is the frequency of occurrrence of the "look for a pattern" strategy contained in grade three mathematics textbooks?
7. What is the frequency of occurrence of the "eliminate possibilities" strategies contained in grade three mathematics textbooks?
8. What is the frequency of occurrence of the "act it out" strategy contained in grade three mathematics textbooks?
9. What is the frequency of occurrence of the "make a systematic list" strategy contained in grade three mathematics textbooks?

Main Findings

Data obtained from the analysis of twenty-four sub-sample units, and accounting for questions 1-3 that deal with textbook problems yielded results summarized in Table 4.

In all of the three textbooks, the sub-sample units yielded 8,564 activities, that is, problems and exercises with and without problem solving. The totl number of problems (exercises and problems having problem solving) was 1,130, which is 13.2% of the total activities. Taking the textbook problems that provide answers to the first three questions, it was found that:

1. The frequency of occurrence of "one-step" problems contained in the sub-sample units of three grade three mathematics textbooks is 703; that is 62.2% of the total problems.
2. The frequency of occurrence of "two-step" problems contained in the sub-sample units of three grade three mathematics textbooks is 57 (5.1%).
3. The frequency of occurrence of "multi-step" problems contained in the sub-sample units of three grade three mathematics textbooks is zero.

Questions 4-9 deal with process problems, according to Polya's model. In all of the three textbooks, the

Table 4

Frequencies and Percentage of Problems in Sub-Sample Units

of Textbooks I, II, and III

Textbook	Textbook Problems						Process Problems												Total Problems in the Textbooks	Total Activities in the Textbooks
	1 Step		2 Steps		Multi Steps		Guess & Check		Make a Drawing		Look for a Pattern		Eliminate Possibilities		Act it Out		Make a Systematic List			
	F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%		
I	202	60.8	17	5.1	0	0	39	11.8	29	8.7	34	10.3	3	.90	8	2.4	0	0	332	2,785
II	283	62.9	27	6	0	0	23	5.1	34	9.6	69	15.3	3	.67	9	2	2	.45	450	2,540
III	218	62.7	13	3.7	0	0	21	6.03	21	6.03	37	10.6	8	2.3	19	5.5	11	3.2	348	3,239
Total of Problems: Categories	703		57		0		83		84		140		14		36		13		1,130	
Total of Problems: Dimensions					760												370		1,130	
Total Activities																				8,564

sub-sample units yielded 370 process problems, which is 32.8% of the total number of problems. When the dimension of process problems is divided into six problem solving strategies corresponding to questions 4-9, it was found that:

4. The frequency of occurrence of "guess and check" strategy contained in the sub-sample units of three grade three mathematics textbooks is 83, which is 7.35% of the total problems.
5. The frequency of occurrence of "make a drawing" strategy contained in the sub-sample units is 84 (7.4%).
6. The frequency of occurrence of "look for a pattern" strategy contained in the sub-sample units is 140 (12.4%).
7. The frequency of occurrence of "eliminate possibilities" strategy contained in the sub-sample units is 14 (1.2%).
8. The frequency of occurrence of "act it out" strategy contained in the sub-sample units is 36 (3.2%).
9. The frequency of occurrence of "make a systematic list" strategy contained in the sub-sample units is 13 (1.2%).

Supplementary Findings

As the study developed and the data were sorted, it became clear that a second level of analysis could further enlighten the central issue of the study, the occurrence of problem solving strategies in grade three mathematics textbooks. Hence a secondary analysis, by textbooks and by topics, was done (Table 5).

Textbook I:

The total number of problems contained in the sub-sample units of Textbook I is 332. The number of textbook problems is 219 (65.9%), and process problems are 113 (34.0%) in number.

Textbook II:

The total number of problems in Textbook II is 450. There are 310 (68.9%) textbook problems and 140 (31.1%) process problems.

Textbook III:

Of a total number of 348 problems in Textbook III, there are 231 (66.4%) textbook problems and 117 (33.6%) process problems.

Topic By Topic Comparison:

Data gathered from the analysis of twenty-four units revealed the following results when similar topics from each of the three textbooks were compared.

Table 5

Textbook By Textbook Comparison

Textbook	Textbook Problems F	Textbook Problems %	Process Problems F	Process Problems %	Total Problems
I	219	65.9	113	34.0	332
II	310	68.9	140	32.1	450
III	231	66.4	117	33.6	348
TOTALS	760		370		1,130

Numeration: (Table 6)

In Textbook I, there are 63 problems dealing with Numeration; 17 (26.98%) of these are textbook problems and 46 (73.01%) are process problems.

In Textbook II, on the same topic, the total number of problems is 76; the number of textbook problems 30 (39.5%), and the number of process problems 46 (60.5%).

In Textbook III, the total number of problems is 33; number of textbook problems 8 (24.3%) and process problems 25 (75.6%).

Addition:

The total number of problems on the topic Addition in Textbook I is 41. Textbook problems are 39 (95.1%) and process problems are 2 (4.9%).

In Textbook II there is no separate unit on Addition.

The total number of problems in Textbook III is 27. Textbook problems are 23 (85.2%) and the number of process problems is 4 (14.8%) (Table 7A).

Subtraction:

The total number of problems on the topic of Subtraction in Textbook I is 32. Textbook problems are 29 (90.6%), and process problems are 3 (9.4%) in number.

Textbook II has no separate unit on Subtraction.

Textbook III has the total number of problems as 21, with the number of textbook problems being 14 (66.7%), and process problems 7 (33.3%) (Table 7A).

Table 6

Topic By Topic Comparison: Numeration

		Textbook Problems						Process Problems												
Textbook	Topic	1 Step		2 Steps		Multi Steps		Guess & Check		Make a Drawing		Look for a Pattern		Eliminate Possibilities		Act it Out		Make a Systematic List		Total Problems in Unit
		F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%	
I	Numeration	17	26.98	0	0	0	0	24	38.1	0	0	19	30.2	2	3.2	1	1.6	0	0	63
II	Number Review	23	30.3	7	9.2	0	0	0	0	1	1.3	45	59.2	0	0	0	0	0	0	76
III	Numerals to 9999	5	15.2	3	9.1	0	0	1	3.1	0	0	23	69.7	0	0	0	0	1	3.1	33
Total in Categories		45		10				25		1		87		2		1		1		172
Total in Dimensions						55												117		172

Table 7A

Topic By Topic Comparison: Addition -- Subtraction

		Textbook Problems						Process Problems												
Textbook	Topic	1 Step		2 Steps		Multi Steps		Guess & Check		Make a Drawing		Look for a Pattern		Eliminate Possibilities		Act it Out		Make a Systematic List		Total Problems in Unit
		F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%	
I	Addition	36	87.8	3	7.3	0	0	1	2.4	1	2.4	0	0	0	0	0	0	0	0	41
II	NO SEPARATE TOPIC ON ADDITION IN THE TEXT.																			
III	Addition Facts	23	85.2	0	0	0	0	0	0	0	0	1	3.7	0	0	3	11.1	0	0	27
Total in Categories		59		3				1		1		1				3				68
Total in Dimensions						62												8		68
I	Sub-traction	27	84.4	2	6.25	0	0	2	6.25	1	3.1	0	0	0	0	0	0	0	0	32
II	NO SEPARATE TOPIC ON SUBTRACTION IN THE TEXT.																			
III	Sub-traction	14	66.7	0	0	0	0	0	0	0	0	6	28.6	0	0	1	4.8	0	0	21
Total in Categories		41		2				2		1		6				1				53
Total in Dimension						43												10		53

continued...2

Table 7A - continued - Page 2

Textbook	Topic	Textbook Problems: 1 Step F	%	2 Steps F	%	Multi Steps F	%	Process Problems: Guess & Check F	%	Make a Drawing F	%	Look for a Pattern F	%	Eliminate Possibilities F	%	Act it Out F	%	Make a Systematic List F	%	Total Problems in Unit
I	Addition and Subtraction	54	84.4	1	1.6	0	0	0	0	0	0	9	14.1	0	0	0	0	0	0	64
II	Addition and Subtraction	49	56.3	6	6.9	0	0	3	3.5	14	16.1	15	17.3	0	0	0	0	0	0	87
III	Addition and Subtraction	NO SEPARATE UNIT ON ADDITION AND SUBTRACTION IN THE TEXT.																		
Total in Categories		103		7				3		14		24								151
Total in Dimensions						110												41		151

are 8 (22.9%).

Textbook III has 46 problems of which 41 (89.9%) are textbook problems and 5 (10.9%) are process problems (Table 7B).

Geometry: (Table 7C)

The total number of problems in Textbook I in the Geometry unit is 9. Of these 3 (33.3%) are textbook problems and 6 (66.7%) are process problems.

In Textbook II, the total number of problems is 22. Of these 9 (40.9%) are textbook problems and 13 (59.1%) are process problems.

The total number of problems in Textbook III is 17. Of these there is 1 (5.9%) textbook problem and 16 (94.1%) process problems.

Decimals: (Table 7D)

The total number of problems dealing with the topic of Decimals in Textbook I is 28. Of these the number of textbook problems is 26 (92.9%), and process problems are 2 (7.1%) in number.

In Textbook II the total number of problems is 9, and of these 9 (100%) are textbook problems. There is no process problem.

In Textbook III the total number of problems is 39. There are 18 (46.2%) textbook problems and 21 (53.8%) process problems.

Table 7B

Topic By Topic Comparison: Multiplication -- Division

		Textbook Problems						Process Problems												
Textbook	Topic	1 Step		2 Steps		Multi Steps		Guess & Check		Make a Drawing		Look for a Pattern		Eliminate Possibilities		Act it Out		Make a Systematic List		Total Problems in Unit
		F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%	
I	Multiplication	9	30	1	3.3	0	0	0	0	14	46.7	4	13.3	0	0	2	6.7	0	0	30
II	Multiplication	35	66.1	4	7.6	0	0	6	11.3	1	9.9	2	3.8	0	0	5	9.4	0	0	53
III	Multiplication	15	62.5	0	0	0	0	0	0	2	8.3	0	0	0	0	7	29.2	0	0	24
Total in Categories		59		5				6		17		6				14				107
Total in Dimension						64												43		107
I	Division	14	51.9	2	7.4	0	0	6	22.2	4	14.8	0	0	0	0	1	3.7	0	0	27
II	Division*	27	77.2	0	0	0	0	0	0	0	0	0	0	8	22.9	0	0	0	0	35
III	Division	41	89.1	0	0	0	0	0	0	2	4.4	3	6.5	0	0	0	0	0	0	46
Total in Categories		82		2				6		6		3		8		1				108
Total in Dimension						84												24		108

*In Textbook II, Division is taken as a chapter from unit Division. Measurement.

Table 7C

Topic By Topic Comparison: Geometry

Textbook	Topic	Textbook Problems						Process Problems												Total Problems in Unit
		1 Step		2 Steps		Multi Steps		Guess & Check		Make a Drawing		Look for a Pattern		Eliminate Possibilities		Act it Out		Make a Systematic List		
		F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%	
I	Geometry	1	11.1	2	22.2	0	0	0	0	5	55.6	0	0	1	11.1	0	0	0	0	9
II	Geometry	9	40.9	0	0	0	0	0	0	9	40.9	2	9.1	0	0	2	9.1	0	0	22
III	Geometry	0	0	1	5.9	0	0	1	5.9	8	47.1	0	0	0	0	7	41.2	0	0	17
Total in Categories		10		3				1		22		2		1		9				48
Total in Dimension							13												35	48

*Geometry appears with the following topics in the three textbooks:
Textbook I : Geometry and Measurement
Textbook II : Division. Geometry
Textbook III: Geometry and Graphs.

Table 7D

Topic By Topic Comparison: Decimals

Textbook	Topic	Textbook Problems						Process Problems												Total Problems in Unit
		1 Step		2 Steps		Multi Steps		Guess & Check		Make a Drawing		Look for a Pattern		Eliminate Possibilities		Act it Out		Make a Systematic List		
		F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%	
I	Decimals	22	78.6	4	14.3	0	0	0	0	0	0	2	7.2	0	0	0	0	0	0	28
II#	Decimals	9	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9
III	Decimals	18	46.2	0	0	0	0	8	20.5	0	0	0	0	3	7.7	0	0	10	25.7	39
Total in Categories	49	4				8						2		3				10		76
Total in Dimension							53											23		76

#In Textbook II, Decimals are taken as a chapter from the unit on Fractions, Decimals, Geometry.

Discussion

Based on the data collected by analyzing 24 sub-sample units in three grade three mathematics textbooks, the researcher observed the following:

1. Textbook I has 4,042 activities (i.e., exercises and problems with and without problem solving), Textbook II has 3,373 activities, and Textbook III has 5,253 activities (Tables 1, 2, 3). From this information it can be seen that Textbook III contains the highest number of activities as compared to Textbooks I and II. The total number of activities in the sub-sample units for Textbook I was 2,785; for Textbook II 2,540, and for Textbook III 3,239. Again Textbook III contains the highest number of activities in its sub-sample units as compared to Textbooks I and II.
2. The total number of problems (i.e., exercises and problems having problem solving) in the sub-sample units in Textbook I is 332 which is 11.9% of the total number of activities in the sub-sample units. Textbook II has 450 problems, that is 17.7% of the total activities. Textbook III has 348 problems and this represents 10.8% of the total activities in the sub-sample units. From this information it is concluded that Textbook II has the lowest number of activities in its sub-sample units, but contains the highest

percentage of problems, whereas Textbook III has the highest number of activities in its sub-sample units, but the lowest percentage of problems.

3. When the problems are divided into the two main categories namely textbook problems and process problems, it is found that Textbook I has 219 textbook problems which is 65.96% of the total number of problems in it. There are 113 process problems that form 34.1% of the total problems. Textbook II has 310 (68.9%) textbook problems and 140 (31.1%) process problems. Textbook III has 231 (66.4%) textbook problems and 117 (33.7%) process problems. It can be seen from this information that Textbook II has the highest percentage of textbook problems (68.9%), Textbook III has the next highest (66.4%) and Textbook I has the lowest (65.9%). Of the process problems, it is observed that Textbook I has the highest percentage (34.1%), Textbook III is the next highest having 33.7%, and Textbook II has the lowest (31.1%). It is interesting to note that the range of difference in the three textbooks as regard to process problems is not very large, between 0.4% and 2.6%.

From the above information it is concluded that Textbook II has the highest percentage of textbook problems and therefore the lowest percentage of process problems.

Textbook I has the highest percentage of process problems, and so contains the lowest percentage of textbook problems. Textbook III falls in the middle of the two. On the other hand, if we look at the percentage of problems as compared to the total number of activities, we note that Textbook III has the highest number of activities and lowest percentage of problems. When the problems are divided into the two main categories, (textbook and process), and compared with the total number of problems in the sub-sample units, it seems to hold the middle position. From looking at the number of activities as compared to the number of problems (Table 5) in each of the three textbooks, it may be inferred that grade three children in the schools of Ontario are more likely doing computational activities than solving problems in their mathematics program. In other words, children seem to be spending more time practicing basic mathematical operations than learning the problem solving nature of mathematics.

Data also point to the fact that there is no pattern as regards the presentation, proportion and percentage of problem solving strategies that appear in the content structure of grade three mathematics textbooks. It cannot be said that any of the three textbooks analyzed is more appropriate than the other for providing grade three students with maximum practice in problem solving.

From Table 4, it is seen that there is more emphasis on one-step textbook problems than on two-step, and multi-step does not appear at all. This fact leads to the conclusion that lower order thinking has more emphasis than higher order thinking, and simple problems are more dominant than problems requiring two or more operations to solve them.

Considering the topic by topic comparison (Tables 6 - 7D), the major observation is again that there is a lack of any pattern regarding the distribution and presentation of problem solving activities. For example, the topic "Numeration" has the highest percentage of numeration problems in Textbook II (Table 7). Textbook I has the highest number of "Addition" problems (Table 7A). Textbook I has the highest number of problems in "Subtraction" topic (Table 7A); Textbook II has the highest "Addition and Subtraction" problems (Table 7A). Textbook II has the highest number of "Multiplication" problems (Table 7B); Textbook III has the highest frequency of "Division" problems (Table 7B) while the highest number of "Geometry" problems are in Textbook II (Table 7C). Textbook III has the highest number of "Decimals" (Table 7D). Textbook II ranks above the other two as having the highest number of problems in four separate units, namely, Numeration, Addition and Subtraction, Multiplication, and Geometry. Textbook I has the highest number of problems in the Addition and Subtraction units. Textbook III ranks the highest with problems in the Division and Decimal units. Again in a topic analysis Textbook II ranks the highest, having the greater number of topics containing the highest number of problems in them. But this does not necessarily lead to the conclusion that one can rely on this textbook more than the other two, since all topics should be seen as equally important in the mathematics program. Placing a problem or exercise in more than one problem solving strategy, i.e., according to the number of strategies it could be solved with, did not inflate the findings as there were very few such problems or exercises.

There are no available criteria for judging the high or low frequency of occurrence of problem solving strategies. However, from an overall glance at the findings based on this data analysis, it would appear that there is more emphasis, more time and more space devoted to computational activities as compared to problem solving exercises.

Although the Ministry's guideline strongly recommends the development of problem solving skills at the grade three level, the data suggest a pale reflection in the textbooks selected. Russell's study (p. 46) suggests somewhat similar results as regards to problem solving. Although the data in Russell's research (1975) were collected on classroom procedures and the questionnaires dealing with experience, education, and attitude towards teaching mathematics were administered to the teachers and principals of the sample schools, the results show some pattern of the frequency of problem solving similar to the study being reported.

During this study, it came to the investigator's attention that there are ancillary materials, such as Teacher's Guides and Activity Books in grade three classrooms, that provide supplementary activities for problem solving. However, evidence was not collected to indicate how much these materials are used. Such evidence might show increased attention given to problem solving. Presence of such materials in the classrooms shows some improvement in the years 1975-85, as the researchers in Russell's study noted the presence of resource material in words only.

This research that is being reported, however, was confined to mathematics textbooks as they are listed in "Circular 14 Textbooks 1984". The investigator selected arbitrarily every other textbook as listed in the Ministry's document. To put it differently, the researcher looked at the policy document and not into practice. These textbooks being listed in the Ministry's policy means that each textbook has an equal chance of being used by the schools of Ontario. It just happened that two of the sample textbooks (II and III) are the least used textbooks. But then again, there seems to be no significant difference found in the frequency of occurrence of textbook and process problems between Textbook I (one of the most used textbooks) and Textbooks II and III (see Table 5, page 74). Besides, three out of a total of six textbooks representing fifty percent of the sample should give some credence to these findings.

Two new mathematics series being developed by Ginn and Addison Wesley were not yet available in the classrooms while this research was being conducted. It would be interesting to find out if they represent problem solving in a different pattern paying more attention to it. Besides its limitations, this study provides some evidence to support the statement that textbook problems and process problems that demand a variety of problem solving strategies should be placed more often and throughout the content of grade three mathematics textbooks.

Suggestions for Further Research

1. The checklist developed for this research provided insightful and useful information regarding problem solving strategies in the subject matter of grade three mathematics textbooks. The research was time consuming, but very intersting. So this investigator

hopes to conduct similar studies using sample textbooks for other grades (4-12).

2. A similar study using the same checklist can be done on the three textbooks that are listed in "Circular 14", but were not part of the sample for this research.
3. At the present moment, there are no known criteria available for judging high or low frequency of occurrence of problem solving strategies. Developing such criteria is an open area for research and would be a significant contribution to the field of mathematics education.
4. Studies that observe and record time spent by the student in the classroom doing computational activities compared with time spent solving problems are needed.
5. If the educational goal of skill development in problem solving is to be achieved, mathematics educators and textbook writers need to share a well defined notion of problem solving that will then guide the content of mathematics textbooks.
6. The Ginn and Addison Wesley textbooks in preparation could be analyzed for textbook and process problems to see if they are an improvement over the mathematics texts widely used in the schools at this time.

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Appendix A

Checklist for Analysis of Textbook and Process Problems

Textbook:

Unit: p:

				Textbook Problems			Process Problems: Strategies Employed						
Topic	Chapter	Page	Problem	1 Step	2 Steps	Multi Steps	Guess & Check	Look for a Pattern	Make a Drawing	Eliminate Possi-bilities	Act it Out	Make a Systematic List	Any Other

Appendix B

Pre-Analysis By the Researcher -- Textbook I

Topic	Page	Textbook Problems: 1 Step	Textbook Problems: 2 Steps	Textbook Problems: Multi Steps	Process Problems: Guess and Check	Process Problems: Make a Drawing	Process Problems: Look for a Pattern	Process Problems: Eliminate Possibilities	Process Problems: Act It Out	Process Problems: Make a Systematic List	Problems in Unit: Text Book	Problems in Unit: Process	Problems in Unit: Total	Total Activities in Unit
Geometry (Unit 4)	68-77	4				4			4		4	8	12	150
Exploring Division (Unit 9)	165-179	10			8	1					10	9	19	204
Subtraction (Unit 13)	237-251-	27	2		2	1					29	3	31	221
Pre-Analysis By the Researcher -- Textbook II														
Addition & Subtraction (Unit 4)	93-124	49	6		2	12	15				53	29	82	393
Division (Unit 6)	157-188	53	2		1	5	3	1	1	1	55	12	67	342
Multiplication (Unit 8)	221-252	58	4		2	3	1		1		62	7	69	428
Pre-Analysis By the Researcher -- Textbook III														
Numerals to 9999 (Unit 3)	40-59	5	3		1	21				1	8	23	31	351
Division Facts 1 (Unit 8)	140-159	40			1	3					40	4	44	354
Decimals (Unit 14)	260-279	17			7		3			9	17	19	26	305

Appendix C

Post-Analysis By the Researcher -- Textbook I

Topic	Page	Textbook Problems: 1 Step	Textbook Problems: 2 Steps	Textbook Problems: Multi Steps	Process Problems: Guess and Check	Process Problems: Make a Drawing	Process Problems: Look for a Pattern	Process Problems: Eliminate Possibilities	Process Problems: Act It Out	Process Problems: Make a Systematic List	Problems in Unit: Text Book	Problems in Unit: Process	Problems in Unit: Total	Total Activities in Unit
Geometry (Unit 4)	68-77	1				5	3				1	8	9	150
Exploring Division (Unit 9)	165-179	12			4	3			2		12	9	21	204
Subtraction (Unit 13)	237-251-	23	2		2						25	2	27	221
Post-Analysis By the Researcher -- Textbook II														
Addition & Subtraction (Unit 4)	93-124	47	2		2	11	15				49	28	77	393
Division (Unit 6)	157-188	48	1			3	3	1	7		49	14	63	342
Multiplication (Unit 8)	221-252	44	9			7	5				53	12	65	428
Post-Analysis By the Researcher -- Textbook III														
Numerals to 9999 (Unit 3)	40-59	5	3		1		23			1	8	25	33	351
Division Facts 1 (Unit 8)	140-159	41			2		3				41	5	46	354
Decimals (Unit 14)	260-279	18			8			3		10	18	21	39	305

Appendix D

Analysis By the First Coder-- Textbook I

Topic	Page	Textbook Problems: 1 Step	Textbook Problems: 2 Steps	Textbook Problems: Multi Steps	Process Problems: Guess and Check	Process Problems: Make a Draw-ing	Process Problems: Look for a Pattern	Process Problems: Eliminate Possi-bilities	Process Problems: Act It Out	Process Problems: Make a Systematic List	Problems in Unit: Text Book	Problems in Unit: Pro-cess	Problems in Unit: To-tal	Total Activities in Unit
Geometry (Unit 4)	68-77	4				4			4		4	8	12	150
Exploring Division (Unit 9)	165-179	10			7	3					10	10	20	204
Subtraction (Unit 13)	237-251-	28			3	1					28	4	32	221

Analysis By First Coder -- Textbook II

Topic	Page	1 Step	2 Steps	Multi Steps	Guess and Check	Make a Draw-ing	Look for a Pattern	Eliminate Possi-bilities	Act It Out	Make a Systematic List	Text Book	Pro-cess	To-tal	Total Activities in Unit
Addition & Subtraction (Unit 4)	93-124	53	7			15	16				60	31	91	393
Division (Unit 6)	157-188	54	1		1	7	6				55	13	68	342
Multiplication (Unit 8)	221-252	44	5	1		1	1				60	3	63	428

Analysis By First Coder -- Textbook III

Topic	Page	1 Step	2 Steps	Multi Steps	Guess and Check	Make a Draw-ing	Look for a Pattern	Eliminate Possi-bilities	Act It Out	Make a Systematic List	Text Book	Pro-cess	To-tal	Total Activities in Unit
Numerals to 9999 (Unit 3)	40-59	5	3		1		32			1	8	34	42	351
Division Facts 1 (Unit 8)	140-159	39		1		9	3				40	11	51	354
Decimals (Unit 14)	260-279	19			12	2				12	19	26	45	305

Appendix E

Analysis By the Second Coder--Textbook I

Topic	Page	Textbook Problems 1 Step	2 Steps	Multi Steps	Process Problems Guess and Check	Make a Drawing	Look for a Pattern	Eliminate Possibilities	Act It Out	Make a Systematic List	Problems in Unit Text Book	Process	Total	Total Activities in Unit
Geometry (Unit 4)	68-77	25					9				25	9	34	150
Exploring Division (Unit 9)	165-179				7	2					0	9	9	204
Subtraction (Unit 13)	237-251-	39									39	0	39	221

Analysis By the Second Coder -- Textbook II

Topic	Page	Textbook Problems 1 Step	2 Steps	Multi Steps	Process Problems Guess and Check	Make a Drawing	Look for a Pattern	Eliminate Possibilities	Act It Out	Make a Systematic List	Problems in Unit Text Book	Process	Total	Total Activities in Unit
Addition & Subtraction (Unit 4)	93-124	48	10		2	14	15				58	31	89	393
Division (Unit 6)	157-188	52	2		2	3	4	1	6	1	54	17	71	342
Multiplication (Unit 8)	221-252	60	1		2	2	1				61	5	66	428

Analylsis By the Second Coder -- Textbook III

Topic	Page	Textbook Problems 1 Step	2 Steps	Multi Steps	Process Problems Guess and Check	Make a Drawing	Look for a Pattern	Eliminate Possibilities	Act It Out	Make a Systematic List	Problems in Unit Text Book	Process	Total	Total Activities in Unit
Numerals to 9999 (Unit 3)	40-59	2	5	3		3	21			4	10	24	34	351
Division Facts 1 (Unit 8)	140-159	46				2	3				46	5	51	354
Decimals (Unit 14)	260-279	13						21		10	13	31	44	305